General Certificate of Education
June 2007
Advanced Level Examination

MATHEMATICS **MM05**
Unit Mechanics 5

Tuesday 26 June 2007 1.30 pm to 3.00 pm

For this paper you must have:
- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MM05.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\ \text{m s}^{-2}$, unless stated otherwise.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 A particle moves with simple harmonic motion along a straight line. Its maximum speed is $4\,\mathrm{m\,s^{-1}}$ and its maximum acceleration is $100\,\mathrm{m\,s^{-2}}$.

(a) Show that the period of motion is $\frac{2\pi}{25}$ seconds. *(4 marks)*

(b) Find the amplitude of the motion. *(1 mark)*

2 A simple pendulum consists of a particle, of mass m, fixed to one end of a light, inextensible string of length l. The other end of the string is attached to a fixed point. When the pendulum is in motion, the angle between the string and the downward vertical is θ at time t. The motion takes place in a vertical plane.

(a) Show, using a trigonometrical approximation, that for small angles of oscillation the motion of the pendulum can be modelled by the differential equation

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} = -\frac{g}{l}\theta$$

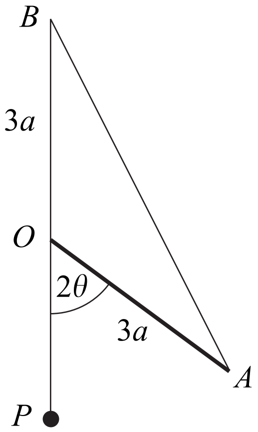
(4 marks)

(b) The pendulum has length 0.5 metres. The pendulum is released from rest with the string taut and at an angle of $\frac{\pi}{400}$ to the vertical.

(i) Given that $\theta = A\cos\omega t$, find the values of A and ω. *(3 marks)*

(ii) Find the maximum speed of the particle in the subsequent motion. *(3 marks)*

3 A uniform rod, OA, of length $3a$ and mass $2m$, is freely pivoted at O. A light, inextensible string, of length $10a$, is attached to the rod at A and passes over a fixed, smooth peg at B, a distance $3a$ vertically above O. A particle, P, of mass m, is attached to the other end of the string. The angle between the rod and the vertical is 2θ, as shown in the diagram.



(a) Show that the total potential energy of the system, V, is given by

$$V = 6mga\cos\theta - 7mga - 3mga\cos 2\theta$$

where gravitational potential energy is taken to be zero at O. *(5 marks)*

(b) Find the **two** values of θ, $0 \leqslant \theta < \frac{\pi}{2}$, for which the system is in equilibrium. *(6 marks)*

(c) Determine the stability of each position of equilibrium. *(4 marks)*

4 A particle of mass m is moving along a smooth wire that is fixed in a plane. The polar equation of the wire is $r = ae^{3\theta}$. The particle moves with a constant angular velocity of 6.

At time $t = 0$, the particle is at the point with polar coordinates $(a, 0)$.

(a) Find the transverse and radial components of the acceleration of the particle in terms of a and t. *(10 marks)*

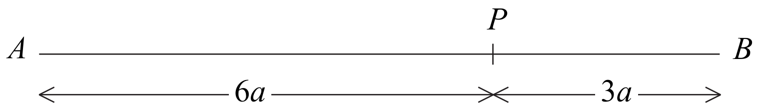
(b) The resultant force on the particle is $\mathbf{F}$.

Show that the magnitude of $\mathbf{F}$, at time t, is $360mae^{18t}$. *(4 marks)*

Turn over ▶

5 The ends of a light, uniform elastic string are fixed to two points, A and B, a distance $9a$ apart on a smooth, horizontal plane. The string is of natural length $6a$ and modulus of elasticity $4mn^2a$, where n is a constant.

A particle of mass m is attached to the string at P, where $AP = 6a$. The natural length of AP is $4a$ and the natural length of BP is $2a$. In this position, the particle is in equilibrium.



The particle is moved a distance $\frac{1}{2}a$ towards B and then released from rest at time $t = 0$. The displacement of the particle from its equilibrium position at time t is x. Hence initially $x = +\frac{1}{2}a$.

The motion of the particle is resisted by a force of magnitude $2mnv$, where v is the speed of the particle at time t.

(a) Show that x satisfies

$$\frac{d^2x}{dt^2} + 2n\frac{dx}{dt} + 3n^2x = 0$$ *(7 marks)*

(b) Given that $n = 1$, find x in terms of a and t. *(8 marks)*

6 A large snowball, which may be modelled as a uniform sphere of radius r, moves with speed v down a slope inclined at $30°$ to the horizontal. The snowball picks up snow at a rate proportional to both its speed and its mass, m, and hence it may be assumed that $\frac{\mathrm{d}m}{\mathrm{d}t} = kmv$ at time t, where k is a constant.

You should ignore any rotational motion of the snowball.

(a) Neglecting any resistance forces acting on the snowball, show that

$$2\frac{\mathrm{d}v}{\mathrm{d}t} + 2kv^2 = g$$ *(4 marks)*

(b) Using the identity

$$\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}x}{\mathrm{d}t} \times \frac{\mathrm{d}v}{\mathrm{d}x} = v\frac{\mathrm{d}v}{\mathrm{d}x}$$

where x is the distance travelled by the centre of the snowball, show that the differential equation in part (a) can be written as

$$2v\frac{\mathrm{d}v}{\mathrm{d}x} = g - 2kv^2$$ *(1 mark)*

(c) At time $t = 0$, $v = 0$ and $x = 0$.

Solve the differential equation in part (b) to find v^2 as a function of x. *(6 marks)*

(d) When $t = 0$, $v = 0$ and $x = 0$, the radius of the snowball is $\frac{1}{3}$ metre.

(i) Show that $r^3 = Ce^{kx}$, where C is a constant to be determined. *(3 marks)*

(ii) Find, in terms of g and k, the speed of the snowball when its radius is 1 metre. *(2 marks)*

END OF QUESTIONS

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